E.3

Energy and Mass Defect

IB PHYSICS | ATOMIC PHYSICS

Unified Atomic Mass Unit

When measuring and reporting the mass of individual atoms and subatomic particles, kilograms are inconveniently large…

The **unified atomic mass unit** is defined as one-twelfth of the mass of an isolated carbon-12 atom

1 mole of Carbon Atoms = 0.012 kg

$$\frac{0.012\ \text{kg}}{6.02 \times 10^{23}} = 1.99 \times 10^{-26}\ \text{kg}$$

$$\frac{1.99 \times 10^{-26}\ \text{kg}}{12} = 1.661 \times 10^{-27}\ \text{kg} = 1\ \text{u}$$

Unified Atomic Mass Unit

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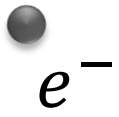
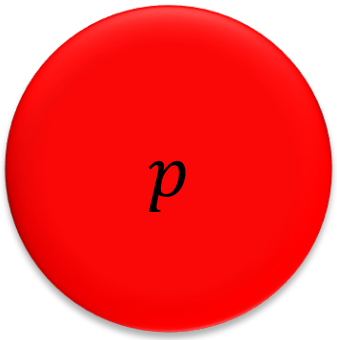
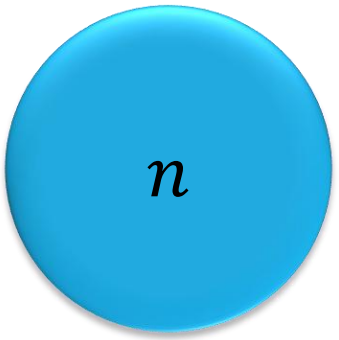
Electron (m_e)	9.110×10^{-31} kg	0.000549 u
Proton (m_p)	1.673×10^{-27} kg	1.007276 u
Neutron (m_n)	1.675×10^{-27} kg	1.008665 u

Unified atomic mass unit	1.661×10^{-27} kg

This is the only time that we will ever use 7 sig figs. In this case, rounding to 1.01 u just wouldn't cut it…







IB Physics Data Booklet

Fundamental constants

Quantity	Symbol	Approximate value
Speed of light in vacuum	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass	m_e	$9.110 \times 10^{-31}\,\mathrm{kg} = 0.000549\,\mathrm{u} = 0.511\,\mathrm{MeV\,c^{-2}}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 1.007276\,\mathrm{u} = 938\,\mathrm{MeV\,c^{-2}}$
Neutron rest mass	m_n	$1.675 \times 10^{-27}\,\mathrm{kg} = 1.008665\,\mathrm{u} = 940\,\mathrm{MeV\,c^{-2}}$
(Unified) atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$

Einstein's Famous Equation

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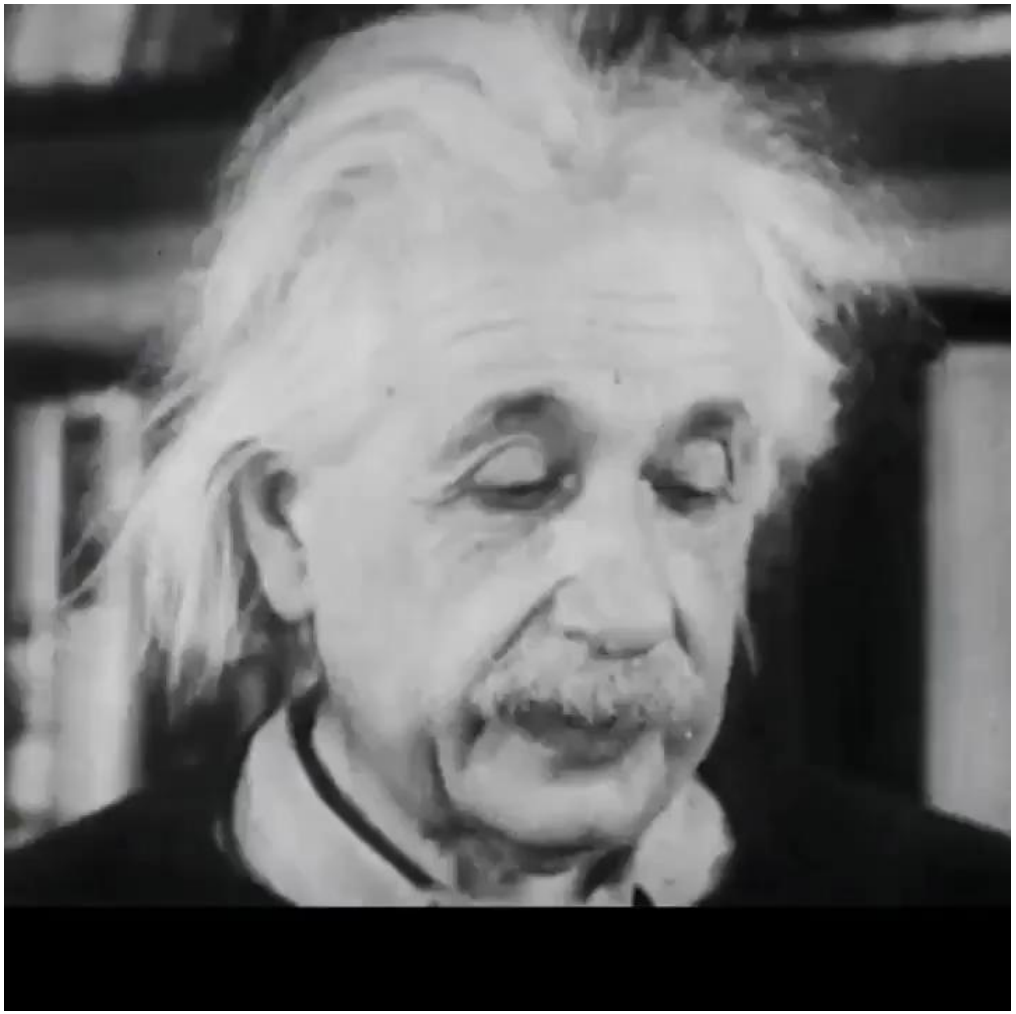

According to Albert Einstein, *"mass and energy are different manifestations of the same things"*

$$E = mc^2$$

Einstein's Famous Equation

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$$E = mc^2$$

Energy [J]

Mass [kg]

Speed of Light $3.00 \times 10^8\ m\ s^{-1}$

What is the energy equivalence of a 5 gram marble?

$$E = (0.005\ kg)(3 \times 10^8\ m\ s^{-1})^2 = 4.5 \times 10^{14}\ J$$

IB Physics Data Booklet

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E. Nuclear and quantum physics

Standard level and higher level	
E.1 Structure of the atom	$E = hf$
E.3 Radioactive decay	$E = mc^2$
E.5 Fusion and stars	$\text{d(parsec)} = \frac{1}{\text{p(arc-second)}}$

$E = mc^2$

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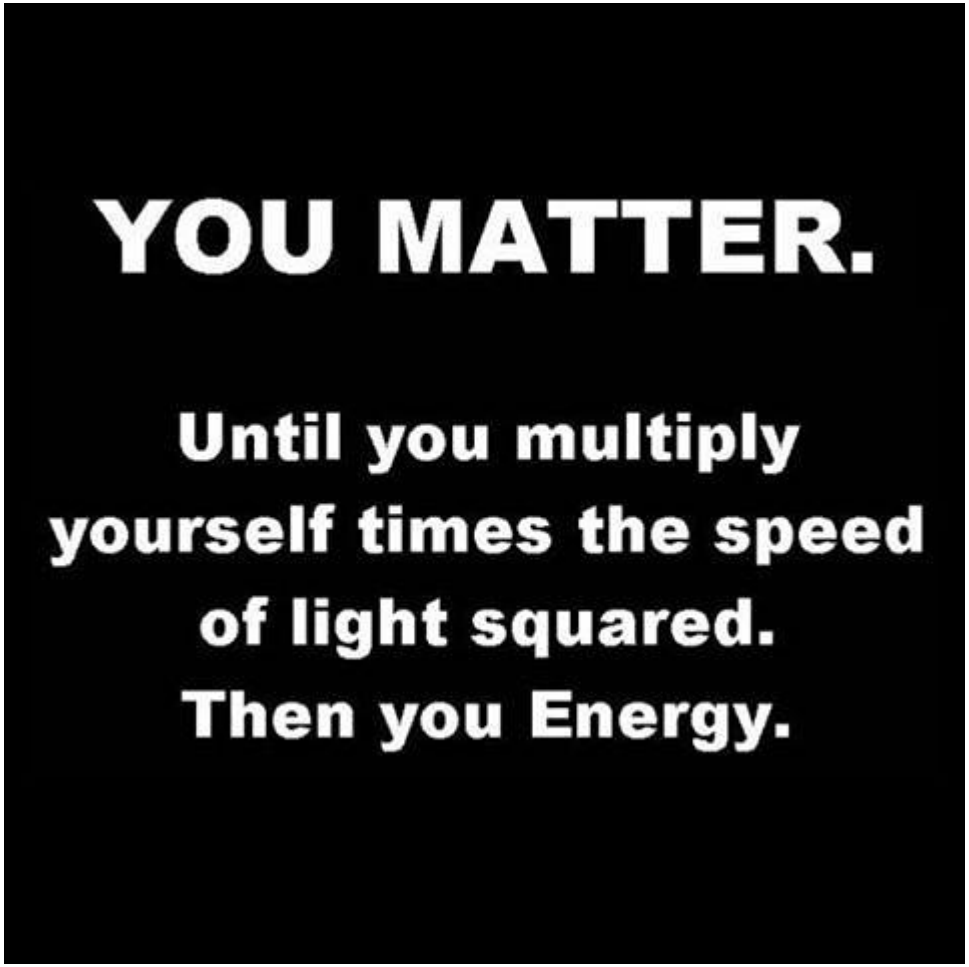
YOU MATTER.
Until you multiply
yourself times the speed
of light squared.
Then you Energy.

Electron-Volts are Back!

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Electron-Volt	eV
1 MeV = 10^6 eV	

$$\{Energy\ in\ eV\} = \frac{\{Energy\ in\ J\}}{elementary\ charge}$$

Speed of Light	**Elementary Charge**	**Atomic Mass Unit**
c = 299,792,458 m s^{-1}	e = 1.6022 × 10^{-19} m s^{-1}	u = 1.66054 × 10^{-27} kg

What is the energy equivalence of 1 atomic mass unit (u)?

$$E = mc^2 = (1.66054 \times 10^{-27})(299{,}792{,}458)^2 = 1.4924 \times 10^{-10}\ \text{J}$$

$$\frac{1.4924 \times 10^{-10}\ \text{J}}{1.6022 \times 10^{-19}} = 931{,}500{,}000\ eV \approx \mathbf{931.5\ MeV}$$

New Unit for Mass

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{MeV}{c^2}$$

$$\boldsymbol{MeV\ c^{-2}}$$

Unified Atomic Mass Unit

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Electron rest mass (m_e)	9.110×10^{-31} kg	0.000549 u	0.511 MeV c^{-2}
Proton rest mass (m_p)	1.673×10^{-27} kg	1.007276 u	938 MeV c^{-2}
Neutron rest mass (m_n)	1.675×10^{-27} kg	1.008665 u	940 MeV c^{-2}
Unified atomic mass unit	1.661×10^{-27} kg	1.000000 u	931.5 MeV c^{-2}

Electron rest mass	m_e	$9.110\times10^{-31}\,\text{kg} = 0.000549\,\text{u} = 0.511\,\text{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673\times10^{-27}\,\text{kg} = 1.007276\,\text{u} = 938\,\text{MeV}\,c^{-2}$
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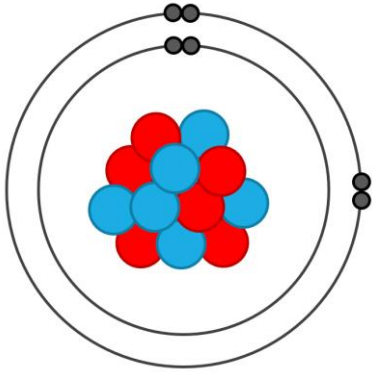
(Unified) atomic mass unit	u	$1.661\times10^{-27}\,\text{kg} = 931.5\,\text{MeV}\,c^{-2}$

Mass of the Nucleus

A neutral Carbon-12 atom contains:

6 protons
6 neutrons
6 electrons



Electron rest mass (m_e)	0.000549 u
Proton rest mass (m_p)	1.007276 u
Neutron rest mass (m_n)	1.008665 u
Unified atomic mass unit	1.000000 u

If the mass of Carbon-12 is defined as exactly 12.00000*u*, then the nucleus mass is:

$$12.00000\text{u} - (6 \times 0.000549\text{u}) = \mathbf{11.996706u}$$

subtract out the electrons

Component Mass

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A nucleus of Carbon-12 contains:

6 protons
6 neutrons

What is the total mass in terms of u?

Electron rest mass (m_e)	0.000549 u
Proton rest mass (m_p)	1.007276 u
Neutron rest mass (m_n)	1.008665 u

Unified atomic mass unit	1.000000 u

6×1.007276 u
6×1.008665 u
} 12.095646 u

Mass Defect | 1+1 > 2

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Mass sum of the Carbon-12 subatomic particles:

$$(6 \times 1.007276\text{u}) + (6 \times 1.008665\text{u}) = 12.095646\text{u}$$

Mass of Carbon-12 nucleus: 11.996706u

Mass Defect ⟶ $12.095646\text{u} - 11.99670\text{u} = \mathbf{0.098946u}$

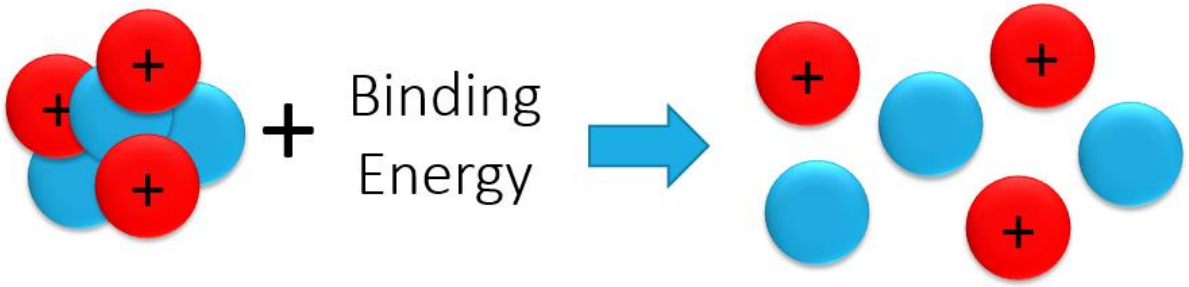
Where did the mass go?

$$E = mc^2$$

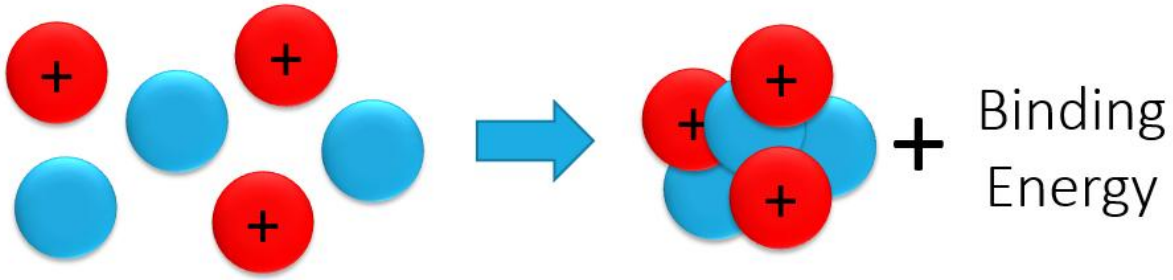
Energy

Binding Energy

Binding Energy is the energy required to separate all of the nucleons



…or the energy released when a nucleus is formed from its nucleons



Mass Defect → Binding Energy

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Unified atomic mass unit	u	1.661×10^{-27} kg	931.5 MeV c^{-2}

$0.098946\text{u} \rightarrow 0.1643 \times 10^{-28}\text{kg} \rightarrow 1.4767 \times 10^{-11}\ \text{J} \rightarrow 92.17 \times 10^{6}\ \text{eV} \rightarrow 92.17\ \text{MeV}$

$\times (1.66054 \times 10^{-27}\ \text{kg})$ **Convert to kg**

$\times 299{,}792{,}428^2$ **E= mc²**

$\div 1.6022 \times 10^{-19}$ **Convert to eV**

Turning u into an equivalent amount of energy in MeV can be a process… ↑↑↑

Luckily, we know how many MeV are contained in 1 u so we can use it as a conversion factor

$$\mathbf{0.098946u} \times \frac{931.5\ MeV\ c^{-2}}{1\ \text{u}} = \boxed{92.17\ MeV\ c^{-2}}$$

$$E = mc^2 = (92.17\ MeV\ \cancel{c^{-2}})(\cancel{c^2}) \quad \boxed{= \mathbf{92.17\ MeV}}$$

Binding Energy per Nucleon

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Binding Energy for Carbon-12 = 92.17 MeV

Nucleons → Particles in the nucleus (protons and neutrons)

Number of Nucleons
for Carbon-12 = **12**

6 protons
6 neutrons

Binding Energy per Nucleon = $\frac{92.17\ MeV}{12}$

$7.68\ MeV\ per\ Nucleon$

Calculate Binding Energy per Nucleon

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Nuclide	# of p	# of n	Nucleus Mass
Iodine-127	53	74	126.87544u

m_e	0.000549u
m_p	1.007276u
m_n	1.008665u
1u	931.5 MeV c^{-2}

Mass Defect

$$\begin{array}{r} 53 \times 1.007276\ \text{u} \\ 74 \times 1.008665\ \text{u} \\ \hline 128.026838\text{u} \end{array} - 126.87544\text{u} = 1.15140\text{u}$$

$$1.15140\ u \times \frac{931.5\ MeV\ c^{-2}}{1\ u} = 1072.53\ MeV\ c^{-2}$$

Convert mass to MeV c^{-2}

$$E = mc^2 = (1072.53\ MeV\ \cancel{c^{-2}})\cancel{c^2} = 1072.53\ MeV$$

$$1072.53\ MeV/127 = \boxed{8.45\ MeV\ per\ Nucleon}$$

Calculate Binding Energy per Nucleon

E.3

*For your assigned nuclide, calculate the binding energy per Nucleon and record data in shared spreadsheet

Use a periodic table to determine atomic # for your element

m_e	0.000549u
m_p	1.007276u
m_n	1.008665u
1u	931.5 MeV c^{-2}

	Element	Nucleus Mass (u)
1	Hydrogen-2	2.013553
2	Helium-3	3.014931
3	Hydrogen-3	3.015500
4	Helium-4	4.001505
5	Lithium-6	6.013476
6	Lithium-7	7.014356
7	Beryllium-9	9.009987
8	Carbon-12	11.996706
9	Nitrogen-14	13.999231
10	Oxygen-16	15.990523
11	Fluorine-19	18.993462
12	Magnesium-24	23.978454
13	Phosphorus-31	30.965527
14	Sulfur-34	33.959083
15	Potassium-39	38.953275

	Element	Nucleus Mass (u)
16	Iron-56	55.920662
17	Arsenic-75	74.903478
18	Krypton-84	83.891734
19	Zirconium-90	89.882739
20	Silver-107	106.879287
21	Tin-120	119.874752
22	Iodine-127	126.875373
23	Cesium-140	139.873608
24	Europium-153	152.886650
25	Tungsten-184	183.910307
26	Gold-197	196.923199
27	Lead-206	205.929447
28	Bismuth-209	208.934833
29	Uranium-235	234.993420
30	Uranium-238	238.000282

Binding Energy per Nucleon

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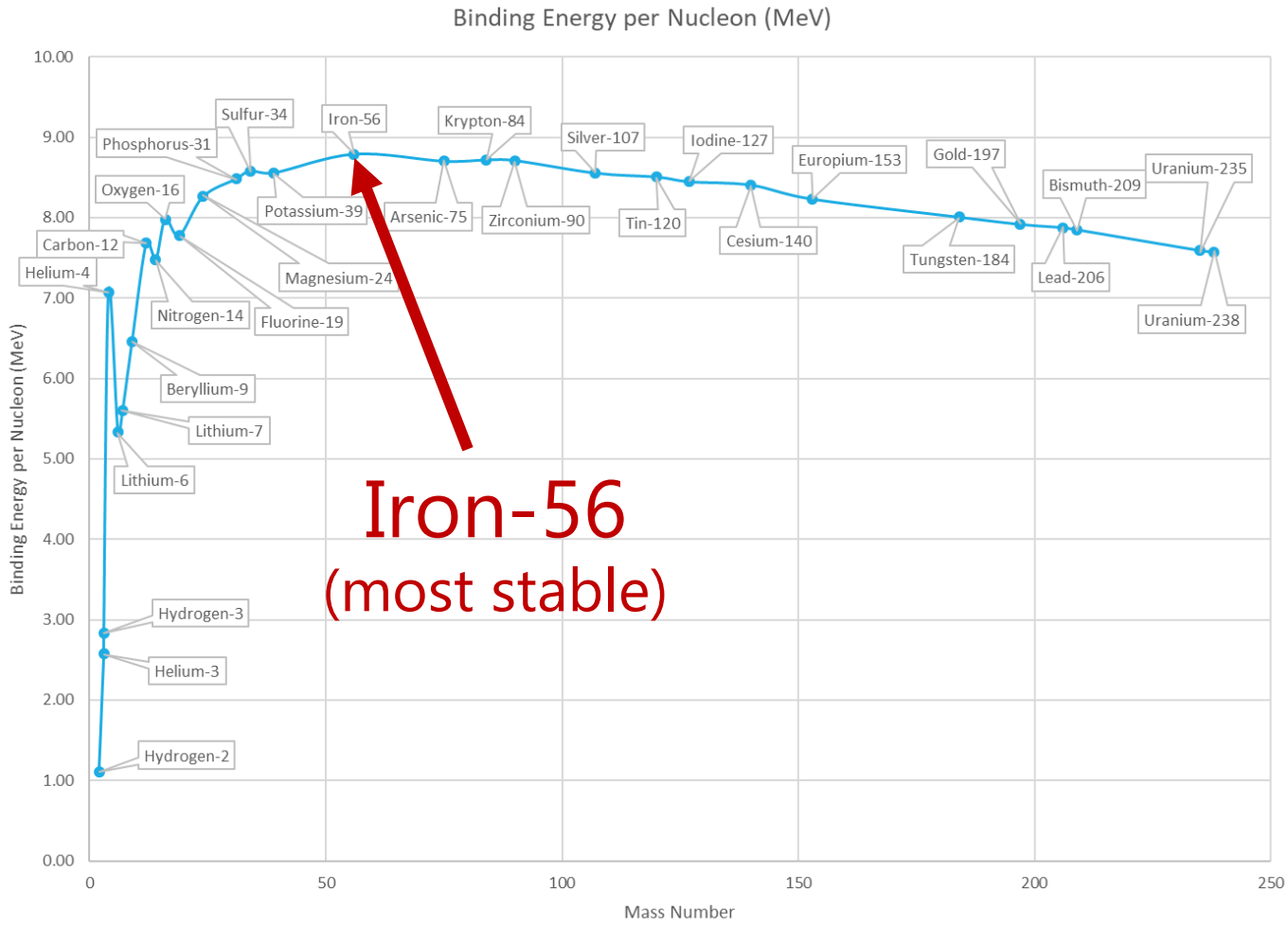

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Lesson Takeaways

- ❑ I can relate units of mass between kilograms (kg) and atomic mass units (u)
- ❑ I can use the mass/energy equivalence to mathematically relate mass and energy
- ❑ I can define mass defect and explain how it is related to energy
- ❑ I can calculate the mass defect of a nuclide
- ❑ I can calculate binding energy from mass defect